Unit VIII: Polarization

Introduction

If we oscillate one end of a string up and down then a transverse wave is generated [see Fig. 22.1(a)]. Each point of the string executes a sinusoidal oscillation in a straight line (along the x-axis) and the wave is, therefore, known as a **linearly polarized wave**. It is also known as a **plane polarized wave** because the string is always confined to the x-z plane. The displacement for such a wave can be written in the form

$$\left.\begin{aligned} & x(z,t)=a\cos(kz-\omega t-\phi_1) \\ \text{and}\quad & y(z,t)=0 \end{aligned}\right\} \qquad (22.1)$$

where a represents the amplitude of the wave and ϕ_1 is the phase constant to be determined from the instant we choose as $t=0$; the y-coordinate of the displacement is always zero. At any instant the displacement will be a cosine curve as shown in Fig. 22.1(a). Further, an arbitrary point $z=z_0$ will execute a simple harmonic motion of amplitude a. The string can also be made to vibrate in the y-z plane [see Fig. 22.1(b)] for which the displacement would be given by

$$\left.\begin{aligned} & x(z,t)=0 \\ \text{and}\quad & y(z,t)=a\cos(kz-\omega t-\phi_2) \end{aligned}\right\} \qquad (22.2)$$

In general, the string can be made to vibrate in any plane containing the z-axis. If one rotates the end of the string on the circumference of a circle then each point of the string will move in a circular path as shown in Fig. 22.2; such a wave is known as a circularly polarized wave and the corresponding displacement would be given by

$$\left.\begin{aligned} & x(z,t)=a\cos(kz-\omega t-\phi) \\ \text{and}\quad & y(z,t)=-a\sin(kz-\omega t-\phi) \end{aligned}\right\} \qquad (22.3)$$

so that x^2+y^2 is a constant ($=a^2$). As we will see later, Eq. (22.3) represents a right circularly polarized wave.

We next consider a long narrow slit placed in the path of the string as shown in Fig. 22.3(a). If the length of the slit is along the direction of the displacement then the entire amplitude will be transmitted as shown in Fig. 22.3(a). On the other hand, if the slit is at right angle to the direction of the displacement, then almost nothing will be transmitted to the other side of the slit [see Fig. 22.3(b)]. This is because of the fact that the slit allows only the component of the displacement, which is along the length of the slit, to pass through; as such, if a longitudinal wave was propagating through the string then the amplitude of the transmitted wave would have been the same for all orientations of the slit. Thus, the change in the amplitude of the transmitted wave with the orientation of the slit is due to the transverse character of the wave. Indeed, an experiment which is, in principle, very similar to the experiment discussed above proves the transverse character of light waves. However, before we discuss the experiment with light waves we must define an unpolarized wave.

We once again consider transverse waves generated at one end of a string. If the plane of vibration is changed in a random manner in very short intervals of time, then such a wave is known as an unpolarized wave*. If an unpolarized wave falls on a slit S_1 (see Fig. 22.4) then the displacement associated with the transmitted wave will be along the length of the slit and a rotation of the slit will not affect the amplitude of the transmitted wave although the plane of polarization of transmitted wave depend on the orientation of the slit (see Fig. 22.4). Thus, the transmitted wave will be linearly polarized and the slit S_1 is said to act as a polarizer. If this polarized beam falls on another slit S_2 (see Fig. 22.4), then by rotating the slit S_2, we obtain a variation of the transmitted amplitude as discussed earlier; the second slit is said to act as an analyzer.

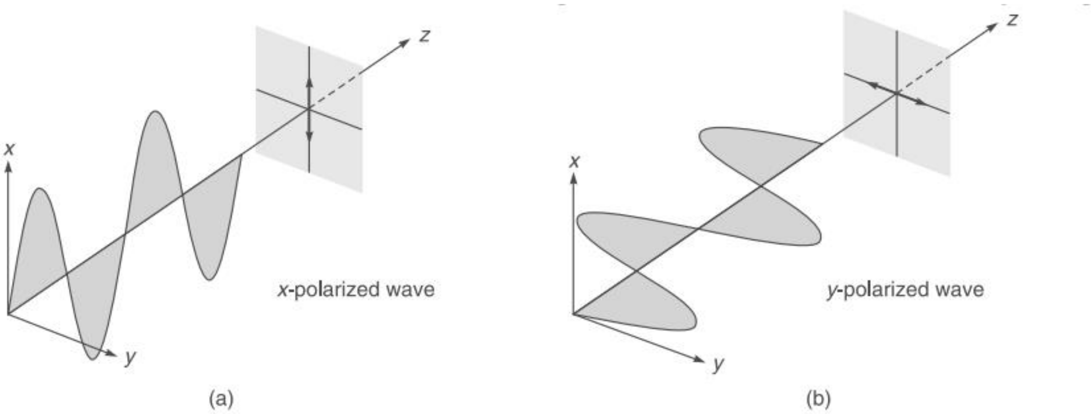


Fig. 22.1 (a) A linearly polarized wave on a string with the displacement confined to the x-z plane; (b) A linearly polarized wave on a string with the displacement confined to the y z plane.

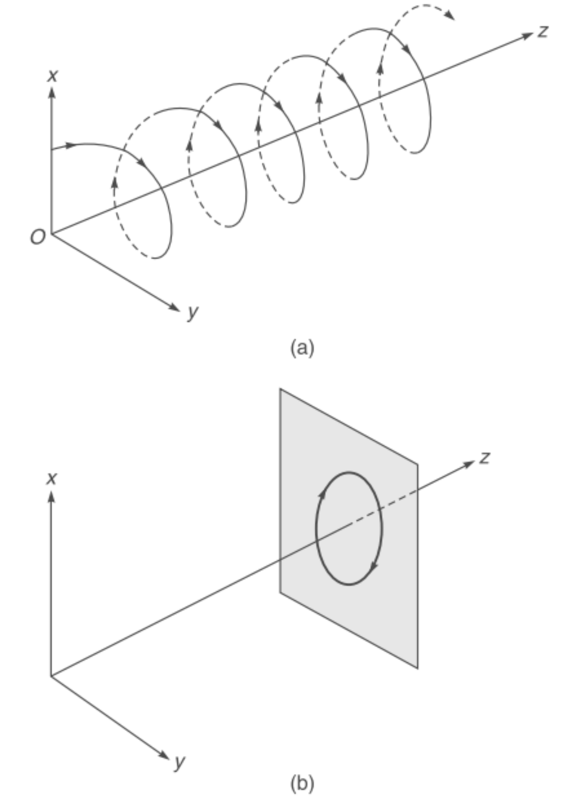


Fig. 22.2 (a) The displacement corresponding to a circularly polarized wave – all points on the string are at the same distance from the z-axis. (b) Each point on the string rotates on the circumference of the circle.

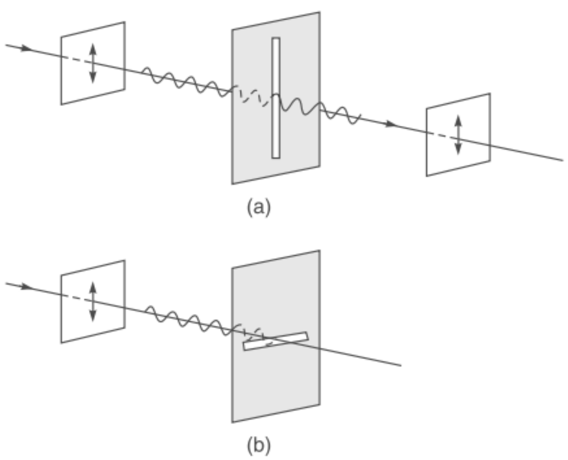


Fig. 22.3 If a linearly polarized transverse wave (propagating on a string) is incident on a long narrow slit, then the slit will allow only the component of the displacement, which is along the length of the slit, to pass through.

Polarized light

The transverse character of light waves was known in the early years of the nineteenth century; however, the nature of the displacement associated with a light wave was known only after Maxwell had put forward his famous electromagnetic theory. We will discuss the basic concept of polarization of light beam in this unit.

Associated with a plane electromagnetic wave there is an electric field **E** and a magnetic field **H** which are at right angles to each other. For a linearly polarized wave propagating in the z direction (in a dielectric) the electric and magnetic fields can be written in the form [see Fig. 22.5]

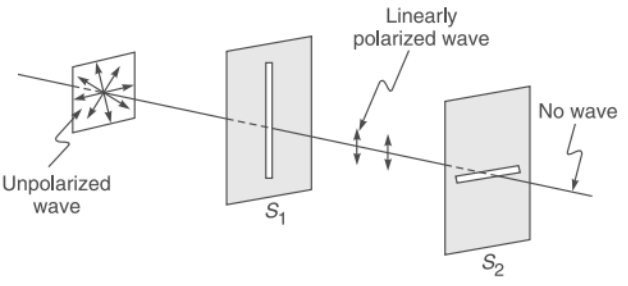


Fig. 22.4 If an unpolarized wave propagating on a string is incident on a long narrow slit S_1, then the transmitted beam will be linearly polarized and its amplitude will not depend on the orientation of S_1. If this polarized wave is allowed to pass through another slit S_2, then the intensity of the emerging wave will depend on the relative orientation of S_2 with respect to S_1.

$$E_x = E_0 \cos(kz - \omega t),\ E_y = 0,\ E_z = 0 \tag{22.4}$$

and $$H_x = 0,\ H_y = H_0 \cos(kz - \omega t),\ H_z = 0 \tag{22.5}$$

where

$$k = \frac{\omega}{v} = \omega\sqrt{\varepsilon\mu} \tag{22.6}$$

and

$$v = \frac{1}{\sqrt{\varepsilon\mu}} \tag{22.7}$$

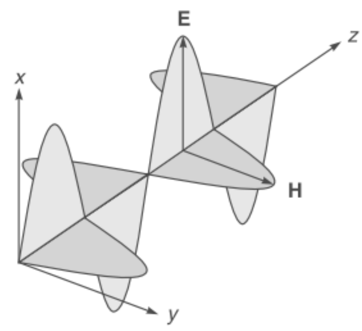
represents the velocity of the waves, ε and μ are the dielectric permittivity and the magnetic permeability of the medium. Since $E_z = 0$ and $H_z = 0$, the wave is transverse. Equations (22.4) and (22.5) also show that **E** and **H** are at right angles to each other and both the vectors are at right angles to the direction of propagation (which is along the z-axis). In fact, the direction of propagation is along the vector ($\mathbf{E} \times \mathbf{H}$). Electromagnetic theory also tells us that for a dielectric [see Sec. 23.3]:

$$H_0 = \frac{k}{\omega\mu} E_0 = \frac{1}{\eta_0 / n} E_0 \tag{22.8}$$

where

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = c\mu_0 \approx 120\pi \quad \text{ohms}$$

is the intrinsic impedance of free space and n is the refractive index of the dielectric (see Sec. 23.3).



x-polarized electromagnetic wave

Fig. 22.5 An x-polarized electromagnetic wave propagating in the z-direction.

We consider an ordinary light beam falling on a Polaroid P_1 as shown in Fig. 22.6; a Polaroid is a plastic-like material used for producing polarized light—it will be discussed in detail in the next section. In general, an ordinary light beam (like the one coming from a sodium lamp or from the sun) is unpolarized, i.e., the electric vector (in a plane transverse to the direction of propagation) keeps changing its direction in a random manner (see Fig. 22.6). When such a beam is incident on a Polaroid the emergent light is linearly polarized with its electric vector oscillating in a particular direction as shown in Fig. 22.6. The direction of the electric vector of the emergent beam will depend on the orientation of the Polaroid. As will be shown in Sec. 22.3.1 the component of **E** along a particular direction gets absorbed by the Polaroid and the component at right angles to it passes through. The direction of the electric vector of the emergent wave is usually called the pass axis of the Polaroid. If the Polaroid P_2 is absent and if the Polaroid P_1 is rotated about the z-axis, there will be no variation of intensity. However, if we place another Polaroid P_2, then by rotating the Polaroid P_2 (about the z-axis) one will observe variation of intensity and at two positions there

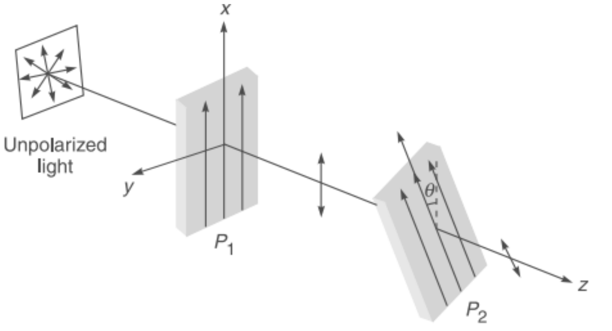


Fig. 22.6 For an unpolarized wave propagating in the +z-direction, the electric vector (which lies in the x-y plane) continues to change its direction in a random manner. If an unpolarized light beam is allowed to fall on a Polaroid, then the emerging beam will be linearly polarized; i.e., the electric vector will oscillate along a particular direction. If we place another Polaroid P_2, then the intensity of the transmitted light will depend on the relative orientation of P_2 with respect to P_1.; if the pass axis of the second polaroid P_2 makes an angle θ with the x-axis, the intensity of the emerging beam will vary as $\cos^2\theta$.

will be almost complete darkness (see Fig. 22.7). A similar phenomenon will also be observed if instead of rotating the Polaroid P_2 we rotate P_1. On the basis of our earlier discussions, this phenomenon proves the transverse character of light; i.e., the displacement associated with a light wave is at right angles to the direction of propagation of the wave. The Polaroid P_1 acts as a polarizer and the transmitted beam is linearly polarized. The second Polaroid acts as an analyzer.

The expression of electric field of a polarized light wave propagating in the z direction can be written as

(i) x polarized light wave

$$E_x = E_o \cos(kz - \omega t), E_y = 0, E_z = 0$$

(ii) y polarized light wave

$$E_x = 0, E_y = E_o \cos(kz - \omega t), E_z = 0$$

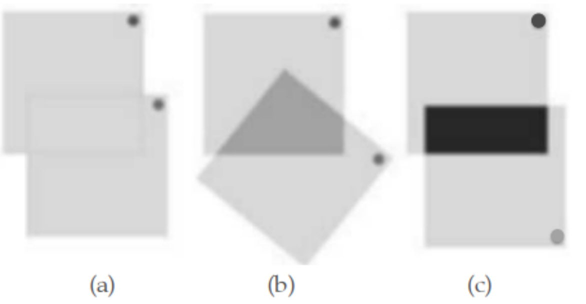
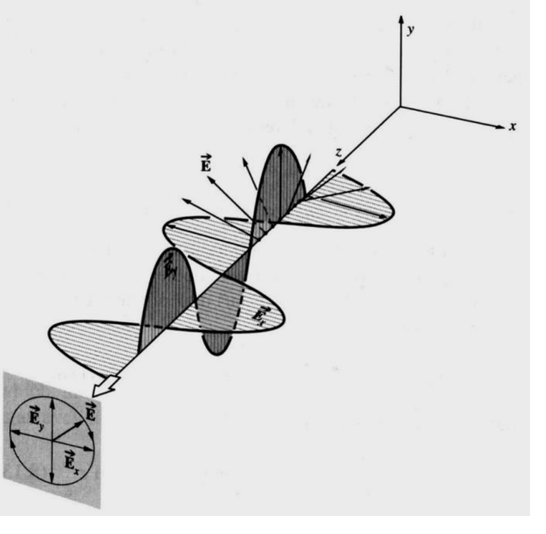


Fig. 22.7 Actual photographs with two Polaroids at different angles of relative orientation. (a) If the two Polaroids are parallel to each other, almost the entire light passes through. (b) When the two Polaroids are oriented at with respect to each other about 50% of the light passes through grey. (c) When the two Polaroids are at right angles to each other (notice the position of the grey dot) almost no light will pass through. Photographs adapted from www.a-levelphysicstutor.com; used with kind permission from Dr. Alan J. Reed. See Fig. 26 in the prelim pages.

(ii) right circularly polarized light wave

$$E_x(z,t) = E_o \cos(kz - \omega t)$$
$$E_y(z,t) = E_o \sin(kz - \omega t)$$

The x and y components of the E-field are 90º out of phase



The resulting E-field rotates **clockwise** around the k-vector (looking along k). This is called a **right-handed** rotation.

(iii) left circularly polarized light wave

$$E_x(z,t) = E_o \cos(kz - \omega t)$$
$$E_y(z,t) = -E_o \sin(kz - \omega t)$$

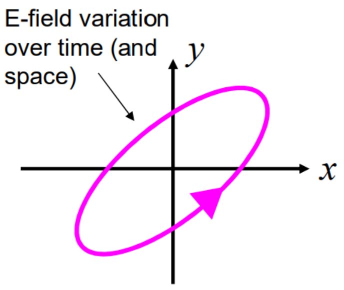
The x and y components of the E-field are always 90º out of phase, but in the other direction.

The resulting E-field rotates **counterclockwise** around the k-vector (looking along k). This is a **left-handed** rotation.

(iv) elliptically polarized light wave

$$E_x(z,t) = E_o \cos(kz - \omega t)$$
$$E_y(z,t) = -E_o \sin(kz - \omega t)$$

where $E_{0x} \neq E_{0x}$



The resulting E-field can rotate clockwise or counter-clockwise around the k-vector.

Malus' Law

Let us consider a Polaroid P_1 which has a pass-axis parallel to the x-axis (see Fig. 22.6); i.e., if an unpolarized beam propagating in the z direction is incident on the Polaroid, then the electric vector associated with the emergent wave will oscillate along the x-axis. We next consider the incidence of the x-polarized beam on the Polaroid P_2 whose pass axis makes an angle θ with the x-axis (see Fig. 22.6). If the amplitude of the incident electric field is E_0, then the amplitude of the wave emerging from the polaroid P_2 will be $E_0 \cos\theta$ and thus the intensity of the emerging beam will be given by

$$I = I_0 \cos^2\theta \qquad \textbf{(22.9)}$$

where I_0 represents the intensity of the emergent beam when the pass axis of P_2 is also along the x-axis (i.e., when $\theta = 0$). Equation (22.9) represents Malus' Law. Thus, if a linearly polarized beam is incident on a Polaroid and if the Polaroid is rotated about the z-axis, then the intensity of the emergent wave will vary according to the above law. For example, if the Polaroid P_2 shown in Fig. 22.6 is rotated in the clockwise direction, then the intensity will increase till the pass-axis is parallel to the x-axis; a further rotation will result in a decrease in intensity till the pass-axis is parallel to the y-axis, where the intensity will be almost zero. If we further rotate it, it will pass through a maximum and again a minimum before it reaches its original position.

Figure 22.7 shows actual photographs of two Polaroids at different relative orientations. In Fig. 22.7 (a) the two are parallel to each other and therefore almost the entire light passes through. In Fig. 22.7 (b) the two Polaroids are oriented at 45° with respect to each other and about 50% of the light passes through; because according to Malus' law $I = I_0 \cos^2 45 = \frac{1}{2} I_0$. Finally, in In Fig. 22.7 (c) the two Polaroids are at right angles to each other (notice the position of the blue dot) and almost no light passes through because $I = I_0 \cos^2 90^\circ = 0$ [see also Fig. 26 in the prelim pages of the book]

Production of polarized light

In this section we will discuss various methods for producing linearly polarized light waves.

1. The wire grid polarizer and the polaroid

The physics behind the working of the wire grid polarizer is probably the easiest to understand. It essentially consists of a large number of thin copper wires placed parallel to each other as shown in Fig. 22.8. When an unpolarized electromagnetic wave is incident on it, then the component of the electric vector along the length of the wire is absorbed. This is due to the fact that the electric field does work on the electrons inside the thin wires and the energy associated with the electric field is lost in the Joule heating of the wires. On the other hand, (since the wires are assumed to be very thin) the component of the electric vector along the x-axis passes through without much attenuation. Thus, the emergent wave is linearly polarized with the electric vector along the x-axis. However, for the system to be effective (i.e., for the E_y component to be almost completely attenuated) the spacing between the wires should be less than λ. Clearly, the fabrication of such a polarizer for a 3 cm microwave is relatively easy because the spacing has to be less than 3 cm. On the other hand, since the light waves are associated with a very small wavelength ($\sim 0.5\ \mu m$), the fabrication of a polarizer in which the wires are placed at distances less than $0.5\ \mu m$ is extremely difficult. Nevertheless, Bird and Parrish did succeed in putting about 30,000 wires in about one inch; for further details see Refs. 22.1 and 22.2. The details of the procedure for making this wire grating are also discussed in Ref. 22.1. The original work of Bird and Parrish was published in 1950 (see Ref. 22.2).

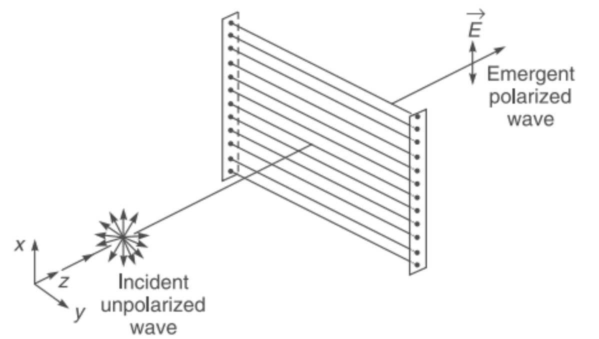


Fig. 22.8 The wire-grid polarizer.

As already pointed out, it is extremely difficult to fabricate a wire grid polarizer which would be effective for visible light. However, instead of long thin wires, one may employ long chain polymer molecules that contain atoms (like iodine) which provide high conductivity along the length of the chain. These long chain molecules are aligned so that they are almost parallel to each other. Because of the high conductivity provided by the iodine atoms, the electric field parallel to the molecules get absorbed. A sheet containing such long chain polymer molecules (which are aligned parallel to each other) is known as a **Polaroid**. When a light beam is incident on such a Polaroid, the molecules (aligned parallel to each other) absorb the component of electric field which is parallel to the direction of alignment because of the high conductivity provided by the iodine atoms; the component perpendicular to it passes through. Thus, the aligned conducting molecules act similar to the wires in the wire grid polarizer and since the spacing between two adjacent long chain molecules is small compared to the optical wavelength, the Polaroid is usually very effective in producing linearly polarized light. The aligning of the long chain conducting molecules is not very difficult and the experimental details of producing the polarizer are given in Ref. 22.1.

2. Polarization by reflection

We consider the incidence of a plane wave on a dielectric; we assume that the electric vector associated with the incident wave lies in the plane of incidence as shown in Fig. 22.9 (a). It will be shown in Sec. 24.2 that if the angle of incidence θ is such that

$$\theta = \theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) \qquad \textbf{(22.10)}$$

then the reflection coefficient is zero. Thus, if an unpolarized beam is incident at this angle, then the reflected beam will be linearly polarized with its electric vector perpendicular to the plane of incidence [see Fig. 22.9(b)]. Equation (22.10) is referred to as the *Brewster's law* and at this angle of incidence, the reflected and the transmitted rays are at right angles to each other; the angle θ_p is known as the polarizing angle or the Brewster angle.

A (commercially available) polarized sunglass blocks the horizontal component and allows only the vertical component to pass through [see Fig. 22.10(a)]. For the air-water interface, $n_1 \approx 1$ and $n_2 \approx 1.33$ and the polarizing angle $\theta_p \approx 53^\circ$. Thus, if the sunlight is incident on the sea at an angle close to the polarizing angle, then the reflected light will be almost linearly polarized [see Fig. 22.10 (b)] and if we now wear polarized sunglasses, the glare, i.e., the light reflected from the water surface, will not be seen. This is the reason why polarized sunglasses are often used by fishermen to remove the glare on the surface and see the fish inside water. Figure 22.11 (a) shows a photograph on the road with ordinary glasses; if we use polarized lenses, the glare can be considerably reduced as shown in Fig. 22.11 (b); see 27 and 28 in the prelim pages. Figure 22.12 shows sunlight incident on a water surface at an angle close to the polarizing angle so that the reflected light is almost polarized. If the Polaroid allows the (almost polarized) reflected beam to pass through, we see the glare from water surface [see Fig. 22.12 (a)]; the glare can be blocked by using a vertical polarizer and one can see the inside of the water [see Fig.

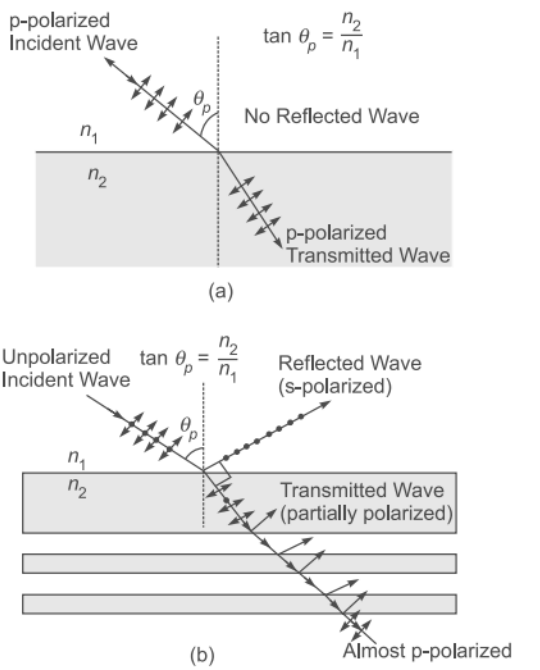


Fig. 22.9 (a) If a p-polarized wave (**E** in the plane of incidence) is incident on the interface of two dielectrics with the angle of incidence equal to q_p (= $\tan^{-1} n_2/n_1$) then the reflection coefficient is zero. (b)If an unpolarized beam is incident at Brewester's angle, the reflected beam is plane polarized whose electric vector is perpendicular to the plane of incidence. The transmitted beam is partially polarized and if this beam is made to undergo several reflections, then the emergent beam is almost plane polarized with its electric vector in the plane of incidence.

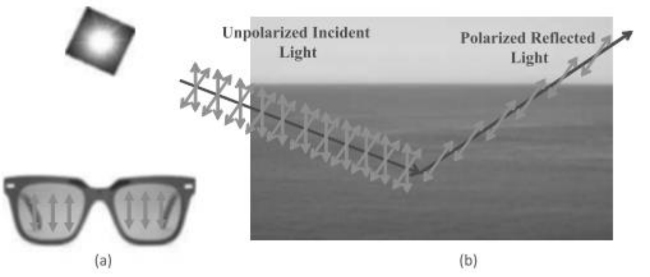


Fig. 22.10 (a) A (commercially available) polarized sunglass blocks the horizontal component and allows only the vertical component to pass through. (b) If the sunlight is incident on the water surface at an angle close to the Brewster angle, then the reflected light will be almost polarized and if we now wear polarized sunglasses, the glare, i.e., the light reflected from the water surface will not be seen. Polarized sunglasses are often used by fishermen to remove the glare on the surface and see the fish inside water. See also Fig. 27 in the prelim pages.

3. Polarization by double refraction

In Sections 22.5 and 22.9 we will discuss the phenomenon of double refraction and will show that when an unpolarized beam enters an anisotropic crystal like calcite, it splits up into two linearly polarized beams (see Fig. 22.13). If by some method, we could eliminate one of the beams, then we would obtain a linearly polarized beam.

A simple method for eliminating one of the beams is through selective absorption; this property of selective absorption is known as dichroism. A crystal like tourmaline has different coefficients of absorption for the two linearly

(a) (b)

Fig. 22.11 (a) A photograph on the road with ordinary glasses. (b) If we use polarized lenses, the glare can be considerably reduced. Photographs adapted from www.esaver.com.my/index.php?option=com_content&view=article&id=95&Itemid=220. See also Fig. 28 in the prelim pages.

polarized beams into which the incident beam splits up. Consequently, one of the beams gets absorbed quickly and the other component passes through without much attenuation. Thus, if an unpolarized beam is passed through a tourmaline crystal, the emergent beam will be almost linearly polarized (see Fig. 22.14).

Another method for eliminating one of the polarized beams is through total internal reflection. We will show in Sections 22.5 and 22.10 that the refractive indices corresponding to the two beams are different. If one can sandwich a layer of a material whose refractive index lies between the two, then for one of the beams, the incidence will be at a rarer medium and for the other it will be at a denser medium. This principle is used in a Nicol prism which

(a) (b)

Fig. 22.12 If the sunlight is incident on the water surface at an angle close to the Brewster angle, then the reflected light will be almost polarized. (a) If the polaroid allows the (almost polarized) reflected beam to pass through, we see the glare from water surface. (b) The glare can be blocked by using a vertical polarizer and one can see the inside of the water. Figure adapted from the website http://polarization.com/water/water.html created by Dr J Alcoz; used with permission of Dr. Alcoz. A color photo appears as Fig. 29 in the prelim pages.

consists of a calcite crystal cut in such a way that for the beam, for which the sandwiched material is a rarer medium, the angle of incidence is greater than the critical angle. Thus, this particular beam will be eliminated by total internal reflection. Figure 22.15 shows a properly cut calcite crystal in which a layer of Canada Balsam has been introduced so that the ordinary ray undergoes total internal reflection. The extraordinary component passes through and the beam emerging from the crystal is linearly polarized.

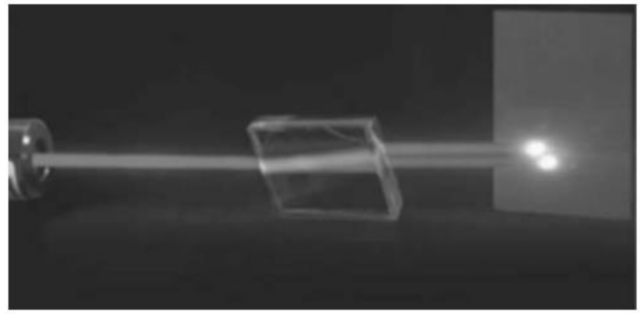


Fig. 22.13 When an unpolarized light beam is incident normally on a calcite crystal, it usually splits up into two linearly polarized beams. Photograph courtesy Professor V Lakshminarayanan and adapted from Ref. 22.16. A color photo appears as Fig. 30 in prelim pages.

4. Polarization by scattering

If an unpolarized beam is allowed to fall on a gas, then the beam scattered at 90° to the incident beam is linearly polarized. This follows from the fact that the waves propagating

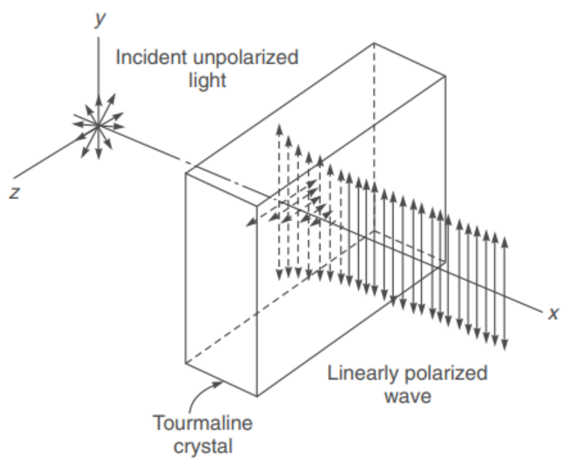


Fig. 22.14 When an unpolarized beam enters a dichroic crystal like tourmaline, it splits up into two linearly polarized components. One of the components gets absorbed quickly and the other component passes through without much attenuation. [Adapted from Ref. 22.3; used with permission.]

in the y direction are produced by the x-component of the dipole oscillations (see Fig. 22.16). The y component of the dipole oscillations will produce no field in the y direction (see Sec. 23.5.1). Indeed, it was through scattering experiments that Barkla could establish the transverse character of X-rays. Clearly, if the incident beam is linearly polarized with its electric vector along the x direction, then there will be no scattered light along the x axis. As such, one can carry out an analysis of a scattered wave by allowing it to undergo a further scattering [see Fig. 22.16(b)].

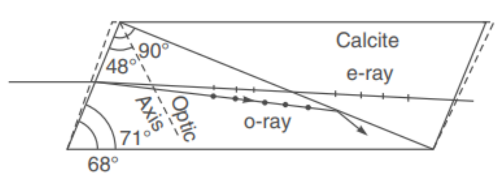


Fig. 22.15 The Nicol prism. The dashed outline corresponds to the natural crystal which is cut in such a way that the ordinary ray undergoes total internal reflection at the Canada Balsam layer.

As discussed in Sec. 7.6 the blue color of the sky is due to Rayleigh scattering of sunlight by molecules in our atmosphere. When the sun is about to set, if we look vertically upwards, light will have a high degree of polarization; this is because the angle of scattering will be very close to 90°. If we view the blue sky (which is vertically above us) with a rotating Polaroid, we will observe considerable variation of intensity.

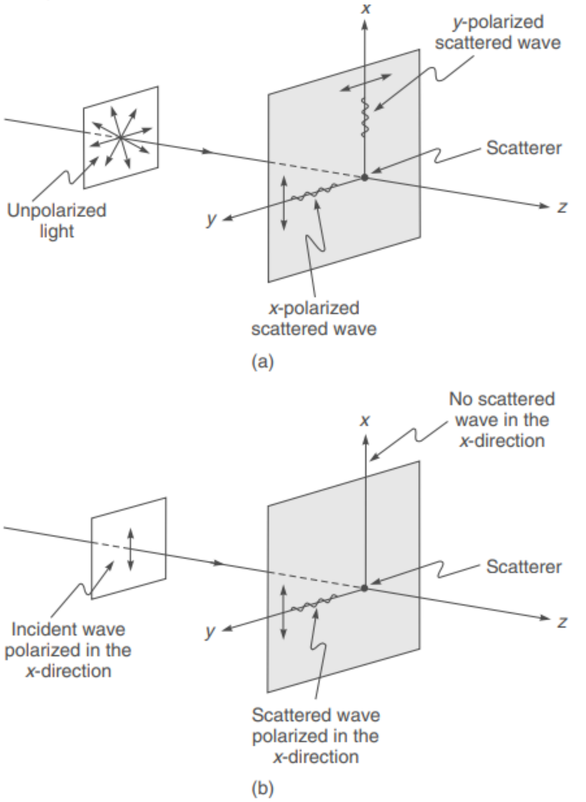


Fig. 22.16 (a) If the electromagnetic wave is propagating along the z-direction, then the scattered wave along any direction perpendicular to the z-axis will be linearly polarized. (b) If a linearly polarized wave (with its **E** oscillating along the x-direction) is incident on a dipole, then there will be no scattered wave in the x-direction.

Polarization by double refraction and Huygen's theory

If an unpolarized light beam is incident normally on a calcite crystal [see Fig. 22.13 and 22.20(a)], it will split up into two linearly polarized beams. The beam which travels undeviated is known as the ordinary ray (usually abbreviated as the o-ray) and obeys Snell's laws of refraction. On the other hand, the second beam, which in general does not obey Snell's laws, is known as the extra-ordinary ray (usually abbreviated as the e-ray). The appearance of two beams is due to the phenomenon of double refraction and a crystal like calcite is usually referred to as a "double refracting" crystal. If we put a polaroid PP' behind the calcite crystal and rotate the polaroid (about NN') then for two positions of the

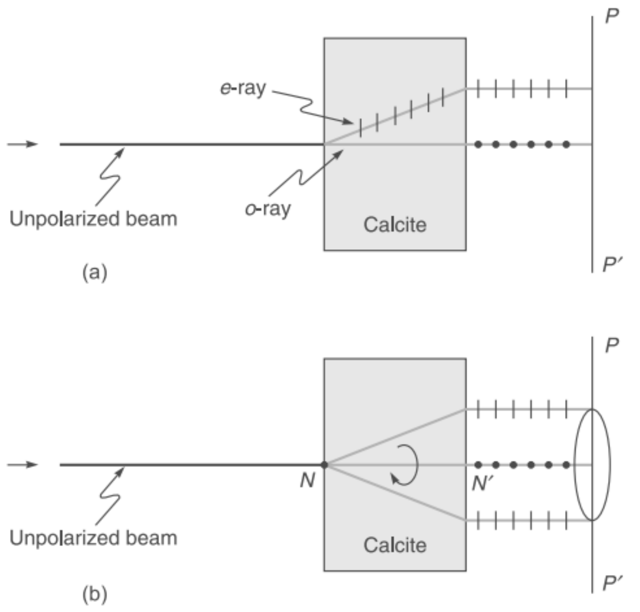


Fig. 22.20 (a) When an unpolarized light beam is incident on a calcite crystal, it usually splits up into two linearly polarized beams. (b) If we rotate the crystal about NN′ then the e-ray will rotate about NN′.

polaroid (when the pass-axis is perpendicular to the plane of the paper) the e-ray will be completely blocked and only the o-ray will pass through. On the other hand, when the pass-axis of the polaroid is in the plane of the paper (i.e., along the line PP') then the o-ray will be completely blocked and only the e-ray will pass through. Further, if we rotate the crystal about NN' then the e-ray will rotate about the axis [see Fig. 22.20 (b)]. Figure 22.21 shows a typical double image as viewed through a doubly refracting crystal like calcite. If we rotate the crystal about the vertical axis, one of the images will be fixed, while the other image will rotate.

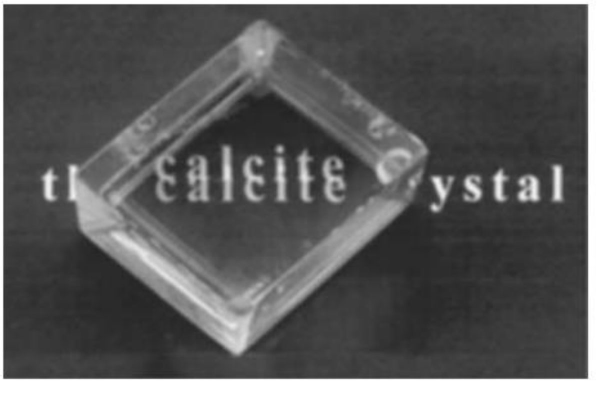


Fig. 22.21 Typical double image of a sentence in a printed text. The ordinary image is fixed, while the upper extraordinary image is shifted and can rotate. Photograph courtesy Professor Vasudevan Lakshminarayanan and adapted from Ref. 22.16; see Fig. 31 in the prelim pages.

In Sec. 22.13 we will show that whereas the velocity of the ordinary ray is the same in all directions, the velocity of the extraordinary ray is different in different directions; a medium (like calcite, quartz), which exhibits different properties in different directions, is called an anisotropic medium. Along a particular direction (fixed in the crystal), the two velocities are equal; this direction is known as the optic axis of the crystal. In a crystal like calcite, the two rays have the same speed only along one direction (which is the optic axis); such crystals are known as uniaxial crystals*. The velocities of the ordinary and the extraordinary rays are given by the following equations [see Eqs. (22.120) and (22.123)]:

$$v_{ro} = \frac{c}{n_o} \qquad \text{(ordinary ray)} \qquad \textbf{(22.36)}$$

$$\frac{1}{v_{re}^2} = \frac{\sin^2\theta}{(c/n_e)^2} + \frac{\cos^2\theta}{(c/n_o)^2} \qquad \text{(extraordinary ray)} \qquad \textbf{(22.37)}$$

where n_o and n_e are constants of the crystal and θ is the angle that the ray makes with the optic axis; we have assumed the optic axis to be parallel to the z-axis. Thus, c/n_o and c/n_e are the velocities of the extraordinary ray when it propagates parallel and perpendicular to the optic axis. Now, the equation of an ellipse (in the z-x plane) is given by

$$\frac{z^2}{a^2} + \frac{x^2}{b^2} = 1 \qquad \textbf{(22.38)}$$

If (ρ, θ) represent the polar coordinates, then $z = \rho\cos\theta$ and $x = \rho\sin\theta$ and the equation of the ellipse can be written in the form

$$\frac{1}{\rho^2} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} \qquad \textbf{(22.39)}$$

In three dimensions, the above equation will represent an ellipsoid of revolution with the optic axis as the axis of revolution. (If we rotate a circle about one of its diameters, we will obtain a sphere, and if we rotate an ellipse about its major (or minor) axis, we will obtain an ellipsoid of revolution). Thus, if we plot v_{re} as a function of θ, we will obtain an ellipsoid of revolution; on the other hand, since v_{ro} is independent of θ, if we plot v_{ro} (as a function of θ), we will obtain a sphere. Along the optic axis, $\theta = 0$ and

$$v_{ro} = v_{re} = \frac{c}{n_o}$$

We next consider the value of v_{re} perpendicular to the optic axis (i.e., for $\theta - \pi/2$). For a negative crystal $n_e < n_o$ and

$$v_{re}\left(\theta = \frac{\pi}{2}\right) = \frac{c}{n_e} > v_{ro} \qquad \textbf{(22.40)}$$

Thus, the minor axis will be along the optic axis and the ellipsoid of revolution will lie outside the sphere [see Fig. 22.22(a)]. On the other hand, for a positive crystal $n_e > n_o$ and

$$v_{re}\left(\theta = \frac{\pi}{2}\right) = \frac{c}{n_e} < v_{ro} \qquad \textbf{(22.41)}$$

The major axis will now be along the optic axis and the ellipsoid of revolution will lie inside the sphere [see Fig. 22.22 (b)]. The ellipsoid of revolution and the sphere are known as the ray velocity surfaces.

We next consider an unpolarized plane wave incident on a calcite crystal. The plane wave will split up into 2 plane

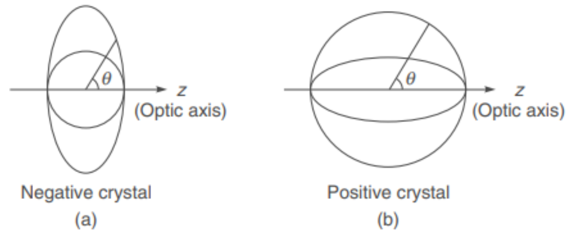


Fig. 22.22 (a) In a negative crystal, the ellipsoid of revolution (which corresponds to the extra-ordinary ray) lies outside the sphere; the sphere corresponds to the ordinary ray. (b) In a positive crystal, the ellipsoid of revolution (which corresponds to the extra-ordinary ray) lies inside the sphere.

waves. One is referred to as the ordinary wave (usually abbreviated as the o-wave) and the other is referred to as the extraordinary wave (usually abbreviated as the e-wave). For both waves, the space and time dependence of the vectors **E**, **D**, **B** and **H** can be assumed to be of the form

$$e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

where **k** denotes the propagation vector and represents the direction normal to the phase fronts. In general, the **k** vector for the o- and e-waves will be different. In Sec. 22.12 we will show that

1. Both ordinary and extraordinary waves are linearly polarized.
2. $\mathbf{D}\cdot\mathbf{k} = 0$ for both o- and e-waves **(22.42)**
   Thus, **D** is always at right angles to **k** and for this reason the direction of **D** is chosen as the direction of "vibrations".
3. If we assume the z-axis to be parallel to the optic axis then,
   $\mathbf{D}\cdot\hat{\mathbf{z}} = 0$ (and $\mathbf{D}\cdot\mathbf{k} = 0$) for the o-wave **(22.43)**
   Thus, for the o-wave, the **D** vector is at right angles to the optic axis as well as to **k**.
4. On the other hand, for the e-wave,
   D lies in the plane containing **k** and the optic axis (and of course, $\mathbf{D}\cdot\mathbf{k} = 0$) **(22.44)**

Using the recipe given above, we will consider the refraction of a plane electromagnetic wave incident on a negative crystal like calcite; a similar analysis can be carried out for positive crystals.

Normal incidence

We first assume a plane wave incident normally on a uniaxial crystal as shown in Fig. 22.23. Without loss of generality, we can always choose the optic axis to lie on the plane of the paper. The direction of the optic axis is shown as a dashed line in Fig. 22.23. In order to determine the ordinary ray, with the point B as the center, we draw a sphere of radius c/n_o. Similarly, we draw another sphere (of the same radius) from the point D. The common tangent plane to these spheres is shown as OO′, which represents the wavefront corresponding to the ordinary refracted ray. It may be noted that the dots show the direction of "vibrations" (i.e., direction of **D**) which are perpendicular to **k** and to the optic axis [see Eq. 22.43].

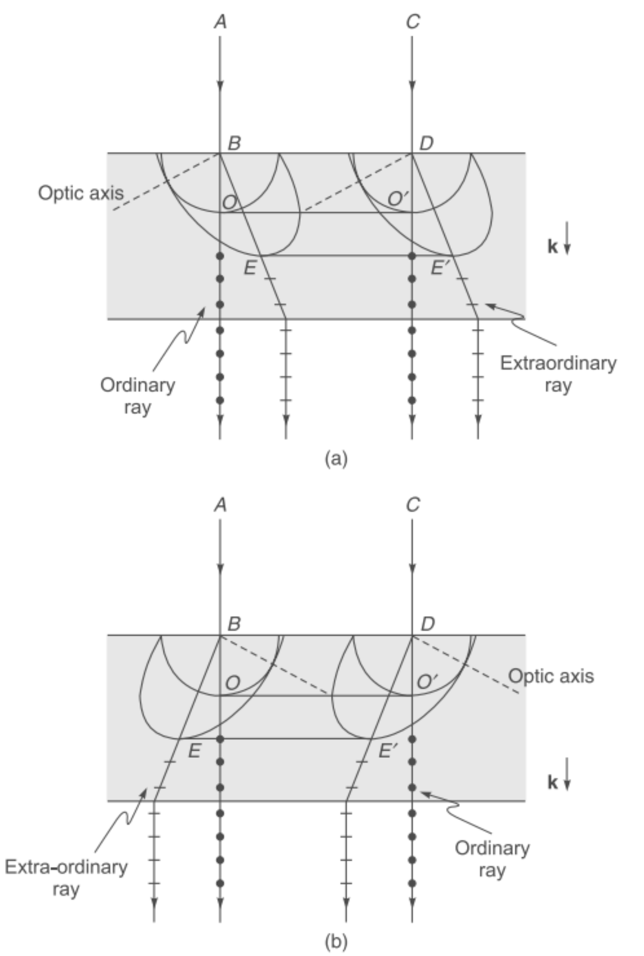


Fig. 22.23 The refraction of a plane wave incident on a negative crystal whose optic axis is along the dashed line.

In order to determine the extraordinary ray, we draw an ellipse (centered at the point B) with its minor axis (= c/n_o) along the optic axis and with major axis equal to c/n_e. The ellipsoid of revolution is obtained by rotating the ellipse about the optic axis. Similarly, we draw another ellipsoid of revolution from the point D. The common tangent plane to these ellipsoids (which will be perpendicular to **k**) is shown as EE′ in Fig. 22.23. If we join the point B to the point of contact O, then corresponding to the incident ray AB, the direction of the ordinary ray will be along BO. Similarly, if we join the point B to the point of contact E (between the ellipsoid of revolution and the tangent plane EE′), then corresponding to the incident ray AB, the direction of the extraordinary ray will be along BE.

It is to be noted that the direction of **k** is the *same* for both *o*- and *e*-waves i.e., both are along BO. However, if we have a narrow beam incident as AB, then while the ordinary ray will propagate along BO, the extraordinary ray will propagate in a different direction BE, as shown in Fig. 22.23(a). Obviously, if we have a different direction of the optic axis [see Fig. 22.23(b)], then, although the direction of the ordinary ray will remain the same, the extraordinary ray will propagate in a different direction. Thus, if a ray is incident normally on a calcite crystal, and if the crystal is rotated about the normal, then the optic axis and the extraordinary ray will also rotate (about the normal) on the periphery of a cone; each time the ray will lie in the plane containing the normal and the optic axis [see Fig. 22.20 (b)].

The ray refractive index corresponding to the extra ordinary ray (n_{re}) will be given by

$$n_{re} = \frac{c}{v_{re}} = \sqrt{n_o^2 \cos^2\theta + n_e^2 \sin^2\theta} \qquad \textbf{(22.45)}$$

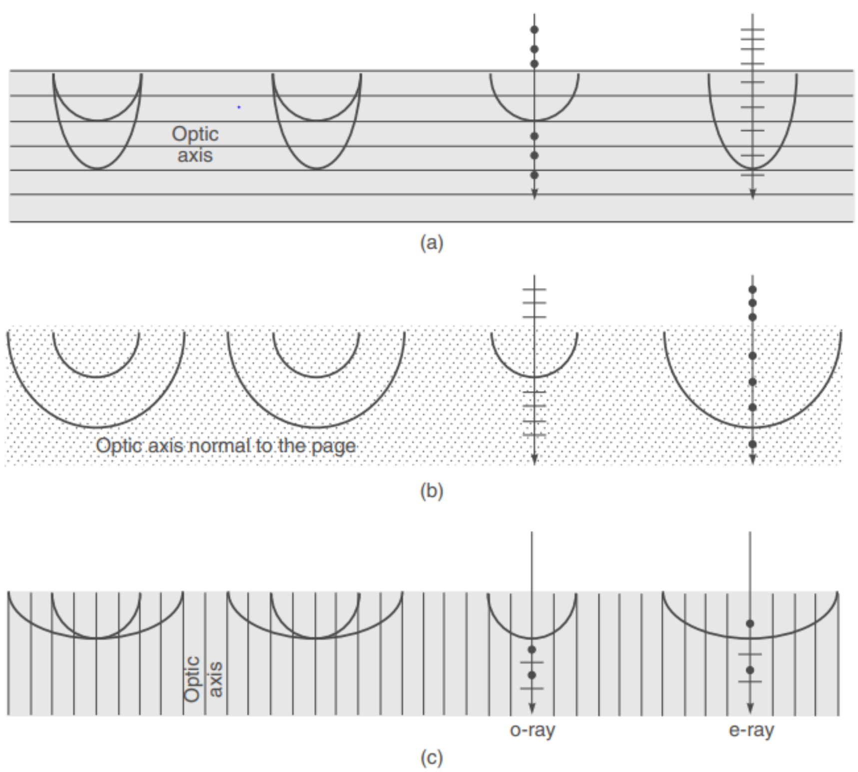


Fig. 22.24 Propagation of a plane wave incident normally on a negative uniaxial crystal. In (a) and (c) the optic axis is shown as parallel straight lines and in (b) the optic axis is perpendicular to the plane of the figure and is shown in dots. In each case, the extra-ordinary and the ordinary rays travel in the same direction.

If one starts with the above equation and uses Fermat's principle to obtain the refracted ray, the results will be consistent with the ones obtained in this section (see Sec. 3.5).

Now, as mentioned earlier, the direction of vibrations for the ordinary ray is normal to the optic axis and the vector **k**; as such, the directions of these vibrations in this case, will be normal to the plane of the paper and have been shown as dots in Fig. 22.23. Similarly, since the direction of vibrations for the extraordinary ray is perpendicular to **k** and lies in the plane containing the extraordinary ray and the optic axis, they are along the small straight lines drawn on the extraordinary ray in Fig. 22.23. Thus, an incident ray will split up into two rays propagating in different directions and when they leave the crystal, we will obtain two linearly polarized beams.

In the above case, we have assumed the optic axis to make an arbitrary angle α with the normal to the surface. In the special cases of $\alpha = 0$ and $\alpha = \pi/2$, the ordinary and the extraordinary rays travel along the same directions as shown in Figs. 22.24(a), (b) and (c). Figure 22.24 (b) corresponds to the case when the optic axis is normal to the plane of the paper; and as such, the section of the extraordinary wavefront in the plane of the paper will be a circle. Once again, both the ordinary and the extraordinary rays travel along the same direction. It may be mentioned that Figs. 22.24 (a) and (b) correspond to the same configuration; in both cases the optic axis is parallel to the surface. The figures represent two different cross-sections of the same set of spherical and ellipsoidal wavefronts.

Now, corresponding to Figs. 22.24 (a) and (b), if the incident wave is polarized perpendicular to the optic axis, it will propagate as an *o*-wave with velocity c/n_o. On the other hand, if the incident wave is polarized parallel to the optic axis, it will propagate as an *e*-wave with velocity c/n_e. In Fig. 22.24 (c) the optic axis is normal to the surface and both waves will travel with the same velocity.

Notice that in the configuration shown in Figs. 22.24 (a) and (b), although both the waves travel in the same direction, they propagate with different velocities. This phenomenon is used in the fabrication of quarter and half wave plates (see Sec. 22.6). On the other hand, in the configuration shown in Fig. 22.24(c), both the waves not only travel in the same direction but they also propagate with the same velocity.

Oblique incidence

We next consider the case of a plane wave incident obliquely on a negative uniaxial crystal [see Fig. 22.25(a)]. Once again we use Huygens' principle to determine the shape of the refracted wavefronts. Let BD represent the incident wavefront. If the time taken for the disturbance to reach the point F from D is t, then with B as center we draw a sphere of radius $(c/n_o)t$ and an ellipsoid of revolution of semi-minor and semi-major axes $(c/n_o)t$ and $(c/n_e)t$ respectively; the semi-minor axis is along the optic axis. From the point F we draw tangent planes FO and FE to the sphere and the ellipsoid of revolution respectively. These planes would represent the refracted wavefronts corresponding to the ordinary and the extraordinary rays respectively. If the points of contact are O and E, then the ordinary and extraordinary refracted rays will propagate along BO and BE respectively; this can also be shown using Fermat's principle (see Sec. 2.5). The directions of vibration of these rays are shown by dots and small lines respectively and are obtained by using the general rules discussed earlier. The shape of the refracted wavefronts corresponding to the particular case of $\alpha = 0$ and $\alpha = \pi/2$ can be obtained very easily. Figure 22.24(b) corresponds to the case when the optic axis is normal to the plane of incidence. The sections of both the wavefronts will be circles; consequently, the extraordinary ray will also satisfy Snell's law and we will have

$$\frac{\sin i}{\sin r} = n_e \quad \text{(for the } e\text{-ray when the optic-axis is normal to the plane of incidence)} \qquad \textbf{(22.46)}$$

Of course, for the ordinary ray we will *always* have

$$\frac{\sin i}{\sin r} = n_o \qquad \textbf{(22.47)}$$

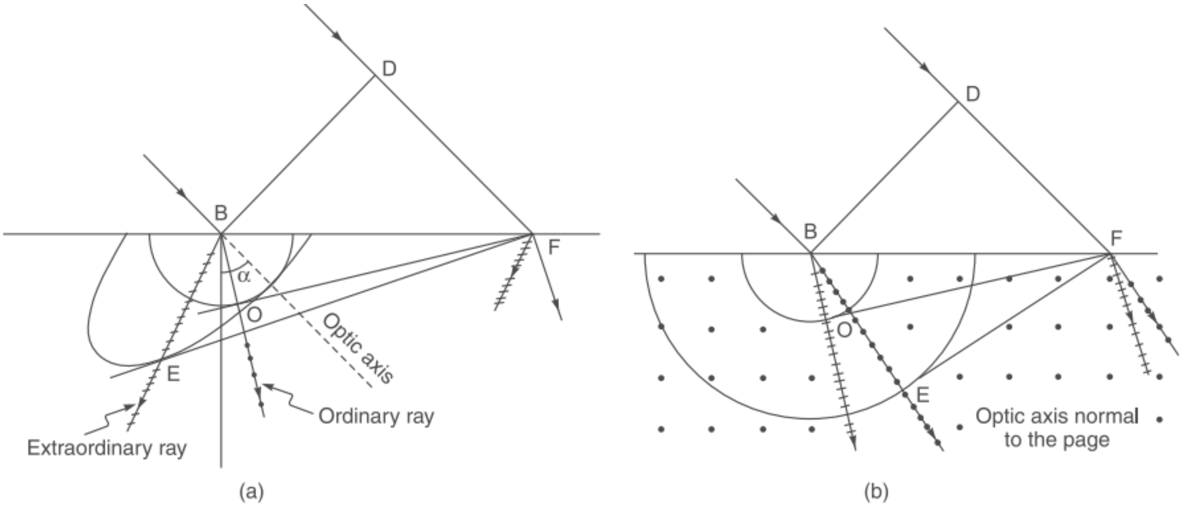


Fig. 22.25 Refraction of a plane wave incident obliquely on a negative uniaxial crystal. In (a), the direction of the optic axis is along the dashed line. In (b), the optic axis is perpendicular to the plane of the paper.

Nicol prism

William Nicol (1770-1851) of Edinburgh developed what is now called the Nicol prism in 1828. The problem with using calcite as a polarizer is the presence of two beams of polarized light. In principle, the E ray can be eliminated by using a narrow crystal, long enough so that the E ray can be sufficiently displaced from the O ray to allow it to be masked off. Nicol used the now classic technique of slicing the crystal diagonally at QS and fastening the two halves back together with a cement (such as canada balsam) of such in index of refraction that the O ray is totally reflected at the internal interface, leaving the E ray to emerge alone from the crystal.

Basic Principle

The basic principle behind Nicol Prism is based on its unique behaviour on the event of incidence of light rays on its surface. When an ordinary ray of light is passed through a calcite crystal, it is broken up into two rays:

- ❖ An 'Ordinary ray' which is polarized and has its vibrations perpendicular to the principle section of the crystal and
- ❖ An extra-ordinary ray which is polarized and whose vibration is parallel to the principle section of the prism. If by some optical means, one of the two rays eliminates, the ray emerging through the crystal will be Plane polarized. In Nicol Prism, ordinary ray is eliminated and Extra-ordinary ray, which is plane polarized, is transmitted through the prism.

Construction

- ❖ It is constructed from the **calcite crystal** ABCD having length three times of its width
- ❖ Its end faces AB and CD are cut such that the angles in the principal section become **68º and 112º in place of 71º and 109º**.
- ❖ The crystal is than cut diagonally into two parts. The surface of these parts are ground to make optically flat and then these are polished.
- ❖ These polished surfaces are connected together with a special cement known as **Canada Balsam.**

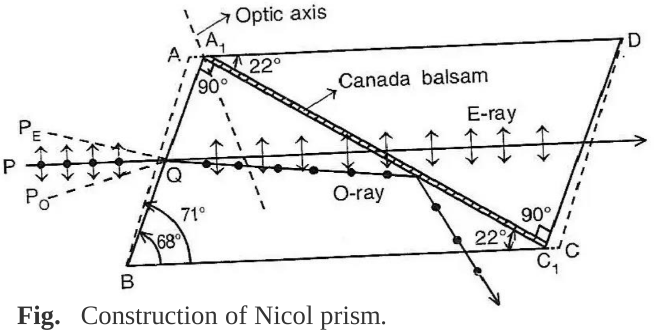


Fig. Construction of Nicol prism.

Working

- ❖ When a beam of unpolarized light is incident on the face A_1B, it gets split into two refracted rays, named O-ray and E-ray.
- ❖ These two rays are plane polarized rays, whose vibrations are at right angle to each other. The refractive index of Canada Balsam cement being 1.55 lies between those of ordinary ($\mu_o = 1.65837$) and extraordinary ($\mu_o = 1.48641$) refractive indices.
- ❖ Thus, the Canada Balsam layer act as an optically rarer medium for the ordinary ray and act as an optical denser medium for the extraordinary ray.
- ❖ When ordinary ray of light travels in the calcite crystal and enters the Canada Balsam cement layer, it passes from denser to rarer medium. Moreover, the angle of incidence is greater than the critical angle, the incident ray is totally internally reflected from the crystal and only the extraordinary ray is transmitted through the prism.
- ❖ Therefore, fully plane polarized light is generated with the help of Nicol prism.

Nicol prism as analyser

Consider two Nicol prisms arranged coaxially one after another. When a beam of unpolarized light is incident on the first prism P, the emergent beam is plane polarized with its vibrations in principal section of first prism. This prism is called polarizer. When principal section of both prisms are parallel, the extraordinary ray from the first prism can freely transmit through the second prism, and thus the intensity of emergent light is maximum.

But when the principal sections are at right angles to each other the intensity of emergent light is minimum i.e., there no light it transmitted through the second prism. This is because the ray on emerging out of the first prism has vibrations in its principal section and, therefore, perpendicular to the principal section of the second prism. It will thus have no component in the principle section of the latter and will travel as an ordinary ray in to be totally reflected at the balsam layer. Here first prism produced plane polarized light and second prism detects and analyses it.

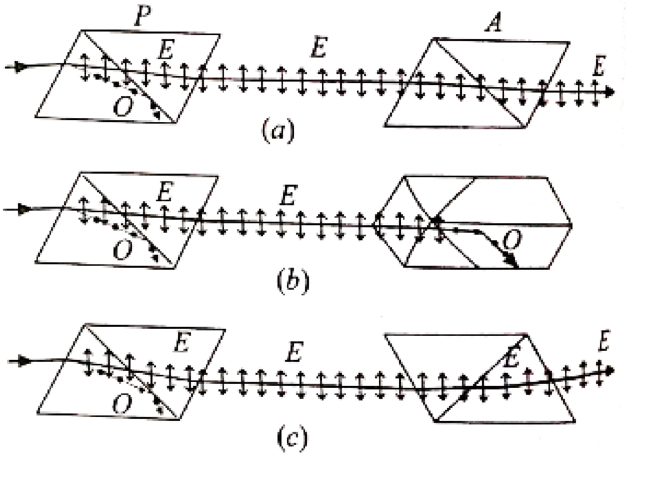


Fig. Two Nicol prisms, one as polarizer (P) and other as analyzer (A) placed in such as way that (a) principal section of both prisms are parallel, (b) principal sections are at right angles and (c) principal sections are at an angle of 180º.

Limitations

When the angle of incidence at the crystal surface is increased, the angle of incidence at Calcite – Balsam surface decreases. When the angle of incidence at the crystal surface becomes greater than 14°, the angle of incidence of Calcite – Balsam surface becomes less than the critical angle. In this position ordinary ray is also transmitted through the prism along with extraordinary ray so light emerging from Nicol prism will not be plane polarized.

When angle of incidence at crystal surface is decreased, the extraordinary ray makes less angle with the optic axis, as a result its refractive index increase, because the refractive index of calcite crystal for E ray is different in different directions through the crystal being maximum when the E ray travels at right angles to the optic axis and minimum when E ray travels along with O ray and no light emerges from the prism

Production of circularly and elliptically polarized light

In the previous section we had considered how a plane wave (incident on a doubly refracting crystal) splits up into two waves each characterized by a certain state of polarization. The direction of vibration associated with the ordinary and extraordinary waves is obtained by using the recipe given by Eqs. (22.42) and (22.43). In this section, we will consider the normal incidence of a plane-polarized beam on a calcite crystal whose optic axis is parallel to the surface of the crystal as shown in Fig. 22.26. We will study the state of polarization of the beam emerging from the crystal. We will assume the y-axis to be along the optic axis. Now, as discussed in the previous section, if the incident beam is x-polarized the beam will propagate as an ordinary wave and the extraordinary wave will be absent. Similarly, if the incident beam is y-polarized the beam will propagate as an extraordinary wave and the ordinary wave will be absent–these are the modes of the crystal. For any other state of polarization of the incident beam, both the extraordinary and the ordinary components will be present. For a negative crystal like calcite $n_e < n_o$ and the e-wave will travel faster than the o-wave; this is shown by putting s (slow) and f (fast) inside the parenthesis in Fig. 22.26. For a positive crystal like quartz $n_e > n_o$ and the e-wave will travel faster than the o-wave.

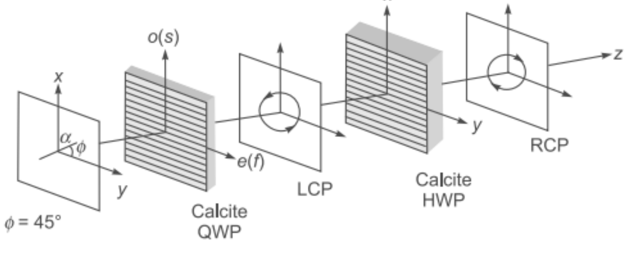


Fig. 22.26 A linearly polarized beam making an angle 45° with the y-axis gets converted to a LCP after propagating through a calcite QWP; further, an LCP gets converted to a RCP after propagating through a calcite HWP. The optic axis in the QWP and HWP is along the y-direction as shown by lines parallel to the y-axis.

Let the electric vector (of amplitude E_0) associated with the incident polarized beam make an angle ϕ with the y-axis; in Fig. 22.26, ϕ has been shown to be equal to 45°—but for the time being we will keep our analysis general and assume ϕ to be an arbitrary angle. Such a beam can be assumed to be a superposition of two linearly polarized beams (vibrating in phase), polarized along the x- and y-directions with amplitudes $E_0 \sin\phi$ and $E_0 \cos\phi$ respectively. The x-component (whose amplitude is $E_0 \sin\phi$) passes through as an ordinary beam propagating with velocity $E_0 \cos\phi$. The y-component (whose amplitude is $E_0 \sin\phi$) passes through as an extraordinary beam propagating with velocity c/n_o. Since $n_e \neq n_o$ the two beams will propagate with different velocities and, as such, when they come out of the crystal, they will not be in phase. Consequently, the emergent beam (which will be a superposition of these two beams) will be, in general, elliptically polarized.

Let the plane $z = 0$ represent the surface of the crystal on which the beam is incident. The x- and y-components of the incident beam can be written in the form

$$\begin{aligned} E_x &= E_0 \sin\phi \cos(kz - \omega t) \\ E_y &= E_0 \cos\phi \cos(kz - \omega t) \end{aligned} \quad (22.48)$$

where $k(=\omega/c)$ represents the free-space wave number. Thus, at $z = 0$, we will have

$$E_x(z=0) = E_0 \sin\phi \cos\omega t; \quad E_y(z=0) = E_0 \cos\phi \cos\omega t$$

Inside the crystal, the x-component will propagate as an ordinary wave (with velocity c/n_o) and the y-component will propagate as an extraordinary wave (with velocity c/n_e)

$$E_x = E_0 \sin\phi \cos(n_o kz - \omega t) \qquad \text{(ordinary wave)}$$

$$E_y = E_0 \cos\phi \cos(n_e kz - \omega t) \qquad \text{(extraordinary wave)}$$

If the thickness of the crystal is d, then at the emerging surface, we will have

$$E_x = E_0 \sin\phi \cos(\omega t - \theta_o)$$

$$E_y = E_0 \cos\phi \cos(\omega t - \theta_e)$$

where $\theta_o = n_o k d$ and $\theta_e = n_e k d$. By appropriately choosing the instant $t = 0$, the components may be rewritten as

$$\begin{aligned} E_x &= E_0 \sin\phi \cos(\omega t - \theta) \\ E_y &= E_0 \cos\phi \cos\omega t \end{aligned} \quad (22.49)$$

$$\text{where} \quad \theta = \theta_o - \theta_e = k\,d\,(n_o - n_e) = \frac{\omega}{c}(n_o - n_e)d \quad (22.50)$$

represents the phase difference between the ordinary and the extraordinary beams. Clearly, if the thickness of the crystal is such that $\theta = 2\pi, 4\pi, 6\pi, \ldots$, the emergent beam will have the same state of polarization as the incident beam. Now, if the thickness d of the crystal is such that $\theta = \pi/2$, the crystal is said to be a quarter wave plate (usually abbreviated as QWP)—a phase difference of $\pi/2$ implies a path difference of a quarter of a wavelength. On the other hand, if the thickness of the crystal is such that $\theta = \pi$, the crystal is said to be a half wave plate (usually abbreviated as HWP).

Example 22.6 As an example, let us consider the case when $\phi = \pi/4$ and $\theta = \pi/2$, i.e., the x- and y-components of the incident wave have equal amplitudes and the crystal introduces a phase difference of $\pi/2$ (see Fig. 22.24). Thus, for the emergent beam we have

$$E_x = \frac{E_0}{\sqrt{2}}\sin\omega t \;;\; E_y = \frac{E_0}{\sqrt{2}}\cos\omega t \qquad (22.51)$$

If we use a method similar to that described in Example 22.1, we will find that a wave described by the above equation represents a left circularly polarized wave. In order to introduce a phase difference of $\pi/2$, the thickness of the crystal should have a value given by the following equation:

$$d = \frac{c}{\omega(n_o - n_e)}\frac{\pi}{2} = \frac{1}{4}\frac{\lambda_0}{(n_o - n_e)} \qquad (22.52)$$

where λ_0 is the free-space wavelength. For calcite, for $\lambda_0 = 5893$ Å and at 18 C, $n_o = 1.65836$ and $n_e = 1.48641$. Substituting these values, we obtain

$$d = \frac{5.893\times10^{-7}}{4\times0.17195}\text{ m} \approx 0.000857\text{ mm}$$

Thus a calcite QWP (at $\lambda_0 = 5893$ Å $= 0.5893$ μm) will have a thickness of 0.000857 mm and will have its optic axis parallel to the surface; such a QWP will introduce a phase difference of $\pi/2$ between the ordinary and the extraordinary components at $\lambda_0 = 5893$ Å. It should be pointed out that if the thickness is an odd multiple of the above quantity, i.e., if

$$d = (2m+1)\frac{\lambda_0}{4(n_o - n_e)}\;;\; m = 0, 1, 2, \ldots \qquad (22.53)$$

then in the example considered above (i.e., when $\phi = \pi/4$), it can easily be shown that the emergent beam will be left circularly polarized for m = 0, 2, 4, … and right circularly polarized for m = 1, 3, 4, …

We next consider the case when the linearly polarized beam (with $\phi = \pi/4$) is incident on a HWP so that $\theta = \pi$, i.e., the x- and y-components of the incident wave have equal amplitudes and the crystal introduces a phase difference of π (see Fig. 22.27). Thus, for the emergent beam we have

$$E_x = -\frac{E_0}{\sqrt{2}}\cos\omega t \;;\; E_y = \frac{E_0}{\sqrt{2}}\cos\omega t$$

which represents a linearly polarized wave with the direction of polarization making an angle of 135° with the y-axis (see Fig. 22.27). If we now pass this beam through a calcite QWP, the emergent beam will be right circularly polarized as shown in Fig. 22.27. On the other hand, if a left circularly polarized is incident normally on a calcite HWP, the emergent beam will be right circularly polarized as shown in Fig. 22.26.

Thus, for a HWP the thickness (for a negative crystal) would be given by

$$d = (2m+1)\frac{\lambda_0}{2(n_o - n_e)}$$

We may mention that if the crystal thickness is such that if $\theta \neq \pi/2, \pi, 3\pi/2, 2\pi, \ldots$ the emergent beam will be elliptically polarized. For a positive crystal (like quartz), $n_e > n_o$ and Eq. (22.49) should be written in the form

$$\begin{aligned} E_y &= E_0 \sin\phi\cos(\omega t + \theta') \\ E_z &= E_0 \cos\phi\cos\omega t \end{aligned} \qquad (22.54)$$

where

$$\theta' = \frac{\omega}{c}d(n_e - n_o)$$

For a quarter wave plate,

$$d = (2m+1)\frac{\lambda_0}{4(n_e - n_o)};\; m = 0,1,2\ldots$$

Thus, if in Fig. 22.26, the calcite QWP is replaced by a quartz QWP, the beam emerging from the QWP will be right circularly polarized.

Example 22.7 We consider a left circularly polarized beam ($\lambda_0 = 5893$ Å $= 0.5893$ μm) incident normally on a calcite crystal (with its optic axis cut parallel to the surface) of thickness 0.005141 mm. The electric field for the incident left circularly polarized beam at z = 0 can be written as

$$E_x = E_1\sin\omega t \;;\; E_y = E_1\cos\omega t \qquad (22.55)$$

Now

$$\theta = \frac{(n_o - n_e)d\times2\pi}{\lambda_0} = \frac{0.17195\times5.141\times10^{-6}\times2\pi}{5.893\times10^{-7}} \approx 3\pi$$

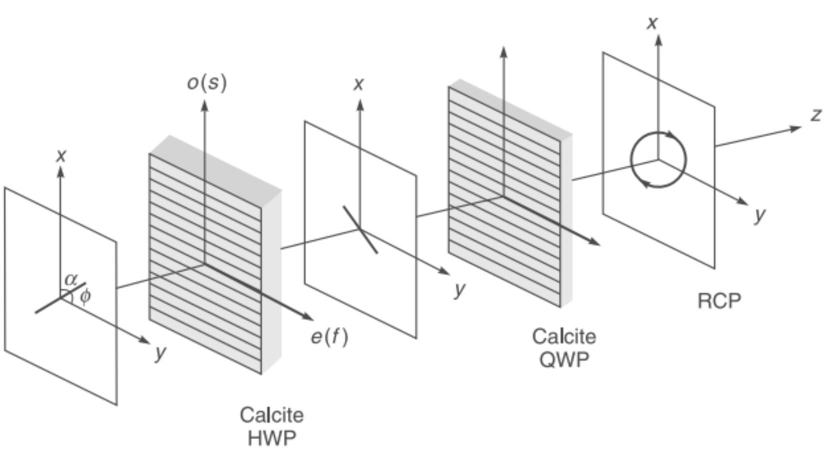


Fig. 22.27 If the linearly polarized beam making an angle 45° with the y-axis is incident on a HWP, the plane of polarization gets rotated by 90°; this beam gets converted to a RCP after propagating through a calcite QWP. The optic axis in the QWP and HWP is along the y-direction as shown by lines parallel to the z-axis.

Thus the emergent wave will be [cf. Eq. (22.49)]

$$E_x = E_1 \sin(\omega t - 3\pi) = -E_1 \sin \omega t \,; \quad E_y = E_1 \cos \omega t$$

which represents a right circularly polarized beam.

Example 22.8 We next consider a left circularly polarized beam ($\lambda_0 = 5893$ Å) is incident on a quartz crystal (with its optic axis cut parallel to the surface) of thickness 0.022 mm. We assume n_o and n_e to be 1.54425 and 1.55336 respectively. The electric field for the incident left circularly polarized beam at $z = 0$ would be given by Eq. (22.55). Further,

$$\theta' = (n_e - n_o)\,\frac{2\pi}{\lambda_0}\,d = 2\pi\,\frac{0.00911 \times 2.2 \times 10^{-5}}{5.893 \times 10^{-7}} \approx 0.68\pi$$

Thus the emergent beam will be

$$E_x = E_1 \sin(\omega t + 0.68\pi) \;;\; E_y = E_1 \cos \omega t$$

which will represent a right elliptically polarized light.

Analysis of polarized light

In the earlier sections we have seen that a plane wave can be characterized by different states of polarizations, which may be anyone of the following:

(a) linearly polarized
(b) circularly polarized
(c) elliptically polarized
(d) unpolarized
(e) mixture of linearly polarized and unpolarized
(f) mixture of circularly polarized and unpolarized
(g) mixture of elliptically polarized and unpolarized light

To the naked eye all the states of polarizations will appear to be the same. In this section, we will discuss the procedure for determining the state of polarization of a light beam.

If we introduce a polaroid in the path of the beam and rotate it about the direction of propagation, then either of the following three possibilities can occur:

(i) If there is complete extinction at two positions of the polarizer, then the beam is linearly polarized.

(ii) If there is no variation of intensity, then the beam is either unpolarized or circularly polarized or a mixture of unpolarized and circularly polarized light. We now put a quarter wave plate on the path of the beam followed by the rotating polaroid. If there is no variation of intensity then the incident beam is unpolarized. If there is complete extinction at two positions, then the beam is circularly polarized (this is due to the fact that a quarter wave plate will transform a circularly polarized light into a linearly polarized light). If there is a variation of intensity (without complete extinction) then the beam is a mixture of unpolarized and circularly polarized light.

(iii) If there is a variation of intensity (without complete extinction) then the beam is either elliptically polarized or a mixture of linearly polarized and unpolarized or a mixture of elliptically polarized and unpolarized light. We now put a quarter wave plate in front of the polaroid with its optic axis parallel to the pass-axis of the polaroid at the position of maximum intensity. The elliptically polarized light will transform to a linearly polarized light. Thus, if one obtains two positions of the polaroid where complete extinction occurs, then the original beam is elliptically polarized. If complete extinction does not occur, and the position of maximum intensity occurs at the same orientation as before, the beam is a mixture of unpolarized and linearly polarized light. Finally, if the position of maximum intensity occurs at a different orientation of the polaroid, the beam is a mixture of elliptically polarized and unpolarized light.

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